Improvement of power distribution network using correlation-based regression analysis

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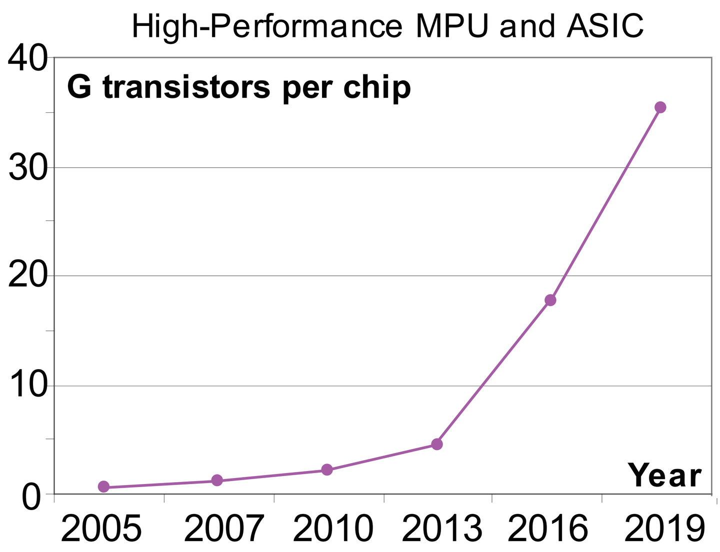
Background

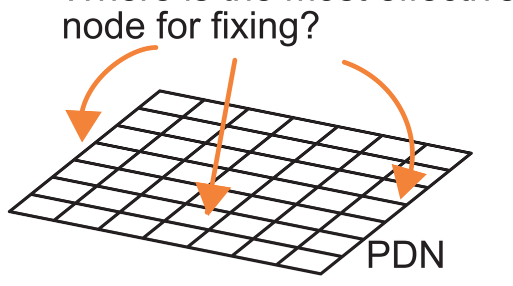
Supply voltage drop has significant impact on timing of logic circuits.

Improvement of PDN is required.

Integration of increasing number of transistors in an LSI makes following issues even more serious



- It requires long time to improve a PDN
- It is difficult to find where to repair
- No quantitative measure to compare PDNs



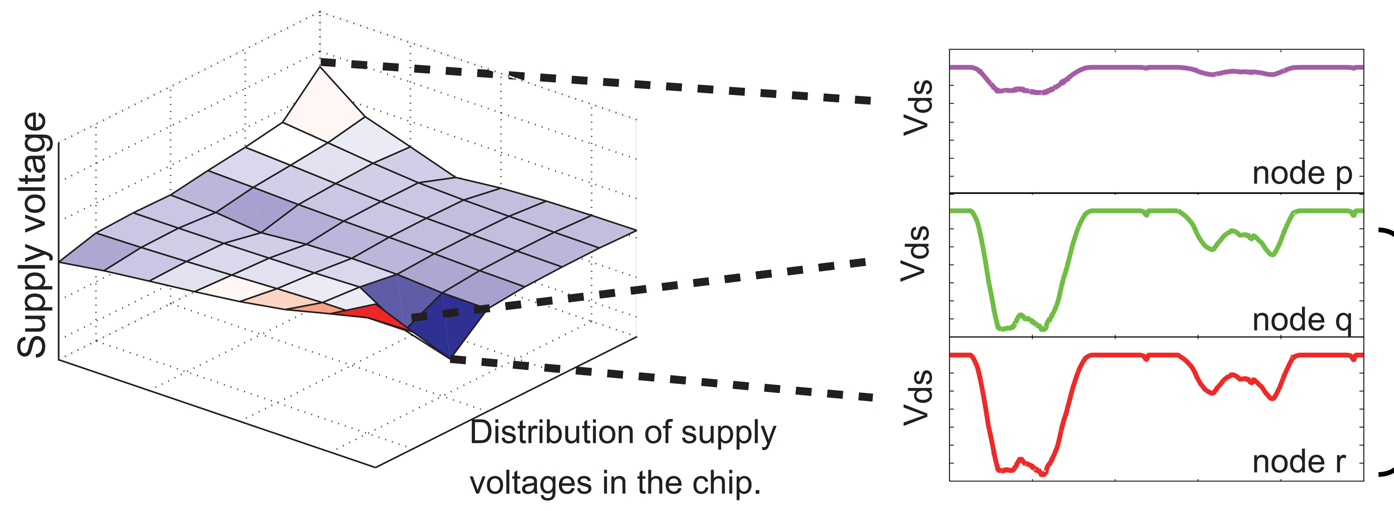
Objective

Statistical approaches are considered to

- propose a measure to quantitatively evaluate the connectivites of the PDN
- propose a procedure which efficiently find appropriate fixing points

Aggregate correlation coefficient

Supply voltages in a chip is temporary and spatially dependent



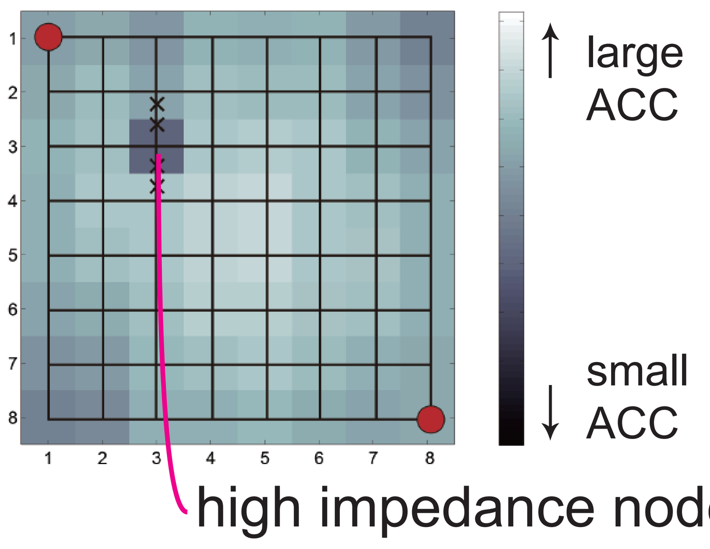
To express the correlation with other nodes by one value, a new metric of aggregate correlation coefficient (ACC) is defined

$$coragg_p = \frac{\sum_{q \in S_n, q \neq p} \mathrm{COR}(\boldsymbol{v}_p, \boldsymbol{v}_q)}{n_{\mathrm{node}} - 1}$$

$\mathrm{COR}(\boldsymbol{v}_p, \boldsymbol{v}_q)$: correlation coefficient between nodes p and q

S_n : a subset of nodes in interest

n_{node} : the number of nodes in S_n



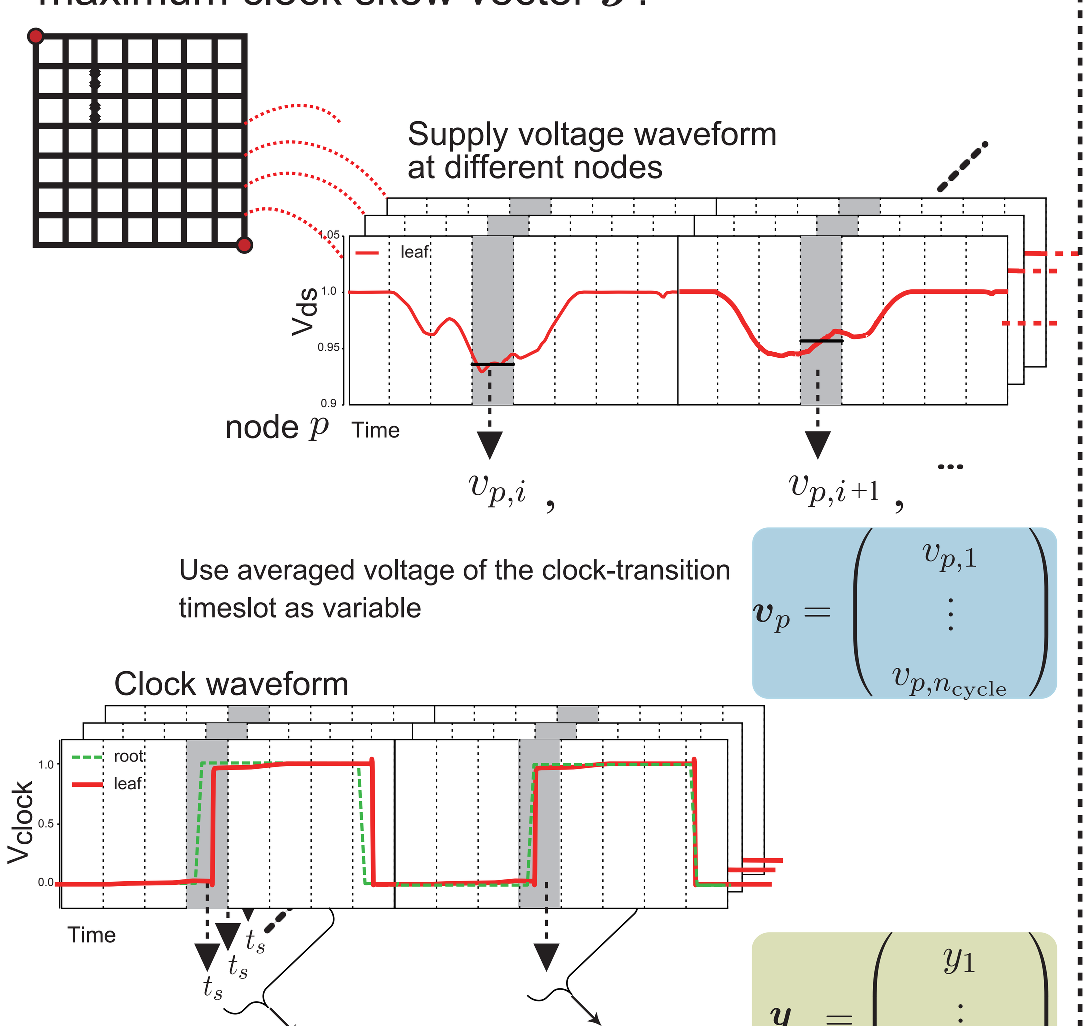
The ACC is a good measure for evaluating connectivities of a node

The ACC quantitates PDN including power consumption distribution

Procedure for PDN fixing

Simulation

Compute effective supply voltages $\boldsymbol{v}$ and maximum clock skew vector $\boldsymbol{y}$.



$v_{p,i}$, $v_{p,i+1}$, …

$$\boldsymbol{v}_p = \begin{pmatrix} v_{p,1} \\ \vdots \\ v_{p,n_{\mathrm{cycle}}} \end{pmatrix}$$

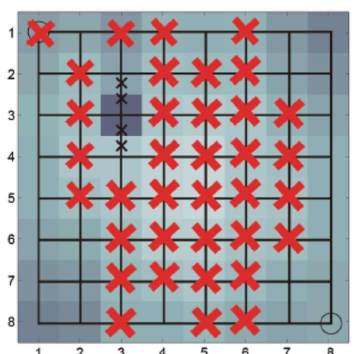
y_i = max(t_s) - min(t_s) , y_{i+1} , … (all nodes, all nodes)

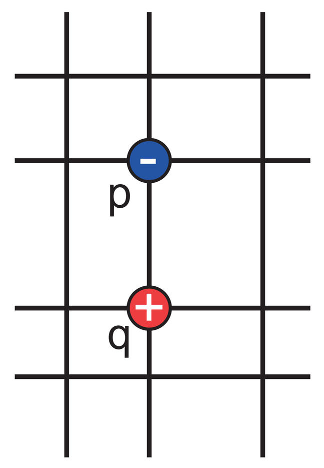
$$\boldsymbol{y} = \begin{pmatrix} y_1 \\ \vdots \\ y_{n_{\mathrm{cycle}}} \end{pmatrix}$$

Vector y is objective variable, such as skew, jitter and so on.

Avoidance of colinearity

Compute ACC to remove large ACC nodes.





Modification at node p

changes voltage of other nodes

Since there are positive / negative coefficient nodes, **all possibility has to be checked to know the best node**

$y = v_p \cdot b_p + v_q \cdot b_q + \cdots$

skew

Sensitivity analysis

Multivariate regression of skew using normalized node voltages

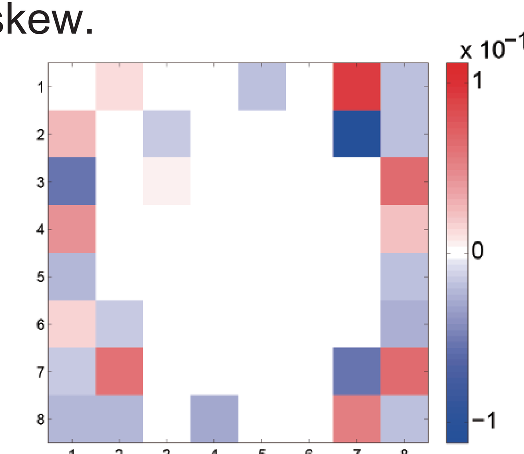
$$\begin{pmatrix} y_1 \\ \vdots \\ y_{n_{\mathrm{cycle}}} \end{pmatrix} = \begin{pmatrix} \cdots & v_{1,n} & \cdots \\ & \vdots & \\ \cdots & v_{n_{\mathrm{cycle}},n} & \cdots \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ \vdots \\ b_p \end{pmatrix}$$

skew, node voltages

$$\boldsymbol{y} = \mathrm{X} \cdot \boldsymbol{b}$$

$$\boldsymbol{b} = (\mathrm{X}^t \mathrm{X})^{-1} \mathrm{X}^t \boldsymbol{y}$$

$\boldsymbol{b}$ is sensitivity of node voltages to skew.



Finding effective points

Estimate skew reduction by

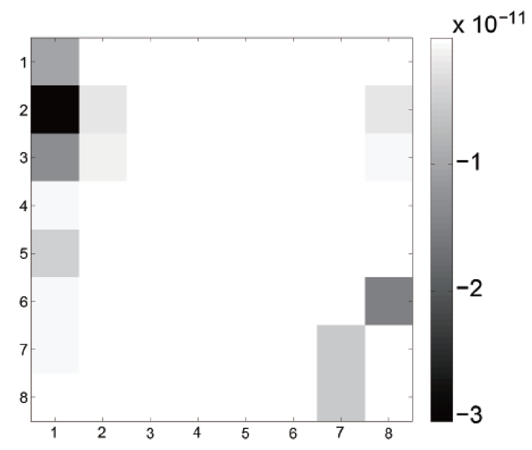
$$\overline{\Delta t_q} = \sum_p \Delta V_{p,q} \cdot b_p$$

where

DC analysis instead of full transient simulation

$$\Delta V_{p,q} = V_{p,q}(DC) - V'_{p,q}(DC)$$

$V_{p,q}(DC), V'_{p,q}(DC)$: Voltage drop of the node p after/before modification of the wires around the node q



An example map of $\overline{\Delta t_q}$

*) When we run DC analysis, the average current of each node is used.

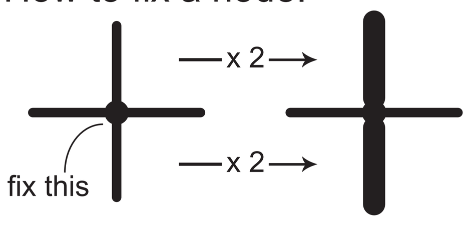
Repeat the procedure until skew satisfies target value or iteration-count limit is reached.

Numerical experiments

We conducted proposed procedure on scaled examples.

size	250μm x 250μm
power	100 mW
clock tree	6-level H-tree

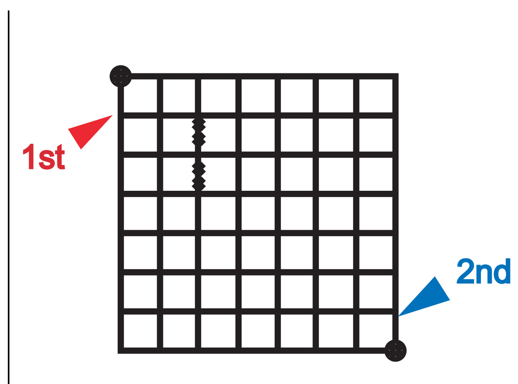
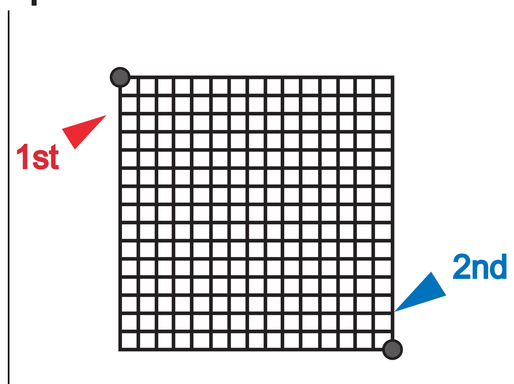
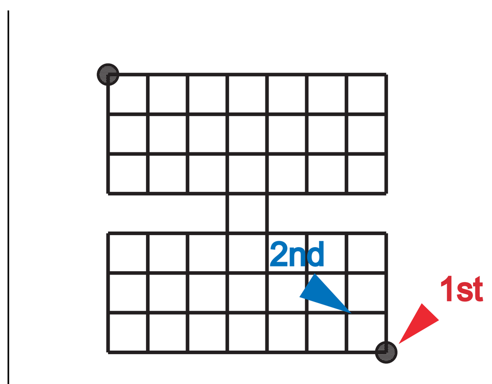
How to fix a node.



● : power supply point

1st : fix node candidate in the first iteration

2nd : fix node candidate in the second iteration

	Case 1	Case 2	Case 3
PDN and fixed nodes			
max skew	13.6 ps → 10.9 ps - 19.8 %	12.8 ps → 12.3 ps - 3.9 %	19.7 ps → 18.6 ps - 5.5 %
time required to find a candidate	49 hours ➤ 9 min	66 hours ➤ 11 min	41 hours ➤ 10 min

Time required to find the most effective node using transient analysis for all candidate nodes.

Conclusion

- The ACC, a new metric which quantitatively evaluate node connection has been proposed. We can find isolated nodes using ACC.
- A procedure which efficiently find an appropriate point to improve PDN is proposed.

Efficient improvement of the given PDN becomes possible